Section 3.8

Derivatives of Inverse Trigonometric Functions

Derivative of the Arcsine

$$y = \sin^{-1} x$$

$$\frac{d}{dx} \sin^{-1} x = \frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}\cos^{-1} x = \frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\csc^{-1}x = \frac{d}{dx}\operatorname{arccsc} x = \frac{-1}{|x|\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{d}{dx}\operatorname{arcsec} x = \frac{1}{|x|\sqrt{x^2 - 1}}$$

$$\frac{d}{dx}\cot^{-1}x = \frac{d}{dx}\operatorname{arccot} x = \frac{-1}{1 + x^2}$$

Example

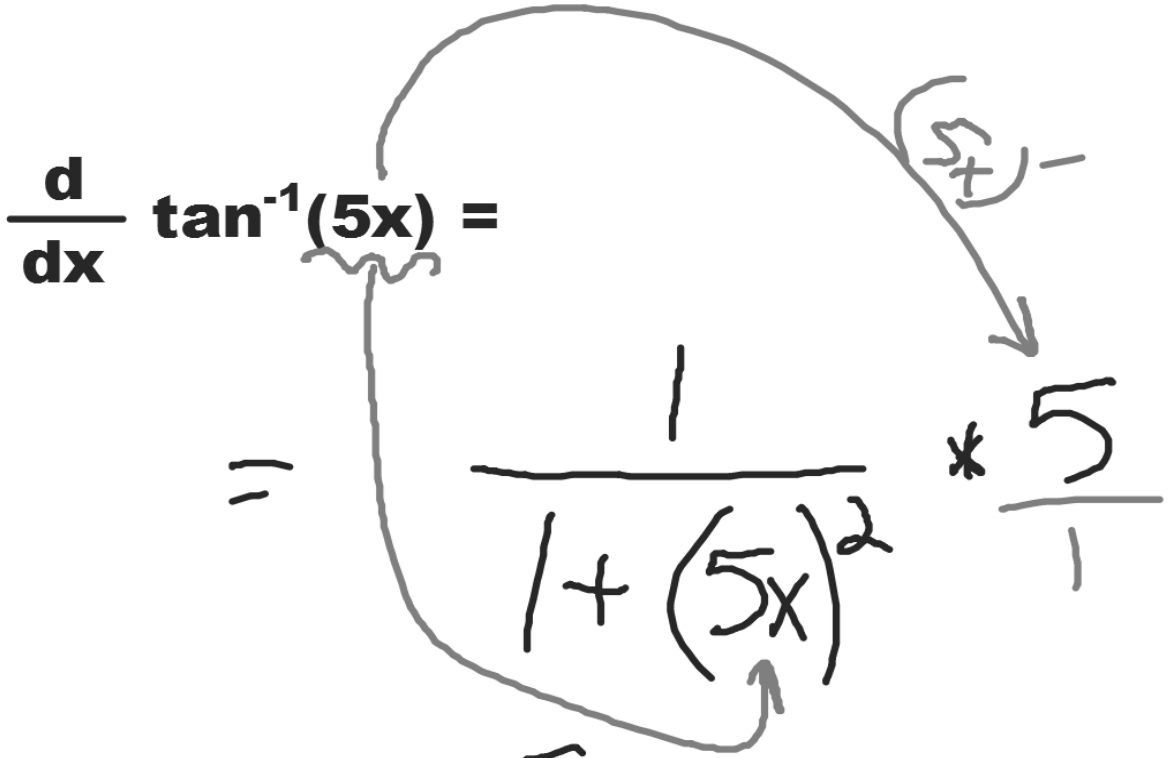
$$\frac{d}{dx}\, 2 \sin^{-1} x =$$

$$= 2 \cdot \frac{d}{dx} \sin^{-1} x$$

$$= 2 \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{2}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}(5x) =$$



$$= \frac{1}{1+(5x)^2} * \frac{5}{1}$$

$$= \frac{5}{1+25x^2}$$

$$\frac{d}{dx}\, x \cos^{-1}(x^2) =$$

$$= (1)(\cos^{-1}(x^2)) + x\left(\frac{-1}{\sqrt{1-(x^2)^2}}\right)\cdot 2x$$

$$= \cos^{-1}(x^2) - \frac{2x^2}{\sqrt{1-x^4}}$$

Find y'

$$y = 2\cos^{-1}(4x) + \tan^{-1} x$$

$$y' = 2\left(\frac{-1}{\sqrt{1-(4x)^2}} * 4\right) + \frac{1}{1+x^2}$$

$$= \frac{-8}{\sqrt{1-16x^2}} + \frac{1}{1+x^2}$$

Find y'

$$y = \frac{\sin^{-1}(x)}{\sqrt{x}}$$

$$y' = \frac{\frac{1}{\sqrt{1-x^2}}\sqrt{x} - \sin^{-1}(x)\frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x})^2}$$

$$= \frac{\frac{\sqrt{x}}{\sqrt{1-x^2}} - \frac{\sin^{-1}(x)}{2\sqrt{x}}}{x}$$

Find y'

$$y = \cos^{-1}x * \tan^{-1}(4x - 1)$$

Find an equation for the line tangent to the graph of $y = \cot^{-1}x$ at $x = -1$.